D 70360 (Pages : 3) Name.................................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Statistics

STS 5B 05—MATHEMATICAL METHODS IN STATISTICS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in one word.*
Each question carries 1 *mark.*

Fill in the blanks :

1. If $s_n = \dfrac{2n+1}{3n+s}$ then $\lim (s_n) =$ ————.
2. Bernoulli's inequality is ————.
3. The sequence (n^2) bounded below by ————.
4. If A is a non-empty bounded subset of R and B is the set of all upper bounds for A. Then inf B = ————
5. A necessary and sufficient condition for a monotone sequence to be convergent is that ————.
6. If $\lim (a_n) = 1$, then $\lim \dfrac{(a_1 + a_2 + \dots + a_n)}{n} =$ ....
7. If $f(x)$ is continuous on $[a, b]$ then there exists a point c in $[a, b]$ such that $\int_a^b f(x)\,dx =$ ....

True *or* False :

8. A bounded function f defined on $[a, b]$ having a finite number of discontinuities is R-integrable over $[a, b]$.
9. The sequence (n^2) is a Cauchy's sequence.
10. Continuity is a necessary condition for the existence of a finite derivative but not a sufficient condition.

(10 × 1 = 10 marks)

Turn over

Section B

*Answer **all** questions.*
Each question carries 2 marks.

11. If $x, y \in R$ then Show that $|x + y| \leq |x| + |y|$.

12. Show that the sequence (s_n) where $s_n = (-1)^n/n$ converges.

13. Prove that if f is continuous at $a \in [a, b]$ then $|f|$ is also continuous at a.

14. Show that if (s_n) is a sequence of non-negative number such that $\lim s_n = |$ then $| \geq 0$.

15. Prove that if x is any positive real number then their exists $n \in N$ such that $x < n$.

16. Examine the validity of Rolle's theorem for the function $f(x) = |x|$ in the interval $[-1, 1]$.

17. Show that a function which is uniformly continuous on an interval I is continuous on that interval.

(7 × 2 = 14 marks)

Section C

*Answer any **three** questions.*
Each question carries 4 marks.

18. State and prove Density theorem.

19. Show that the function $f(x) = 1/x$ is not uniformly continuous on $[0, 1]$.

20. Show that the sequence $\left(\log \frac{1}{n}\right)$ is properly divergent sequence.

21. If $f(x) = (x - 1)(x - 2)(x - 3)$ and $a = 0, b = 4$, find c using Lagrange's mean value theorem.

22. If P is a partition of interval $[a, b]$ and f is a bounded function defined on $[a, b]$. Show that M (b-
$M(b - a) \geq U(P, f) \geq L(P, f) \geq m(b - a)$. Where $M = \sup f$, $m = \inf f$.

(3 × 4 = 12 marks)

Section D

*Answer any **four** questions.*
Each question carries 6 marks.

23. State and Prove Cauchy's mean value theorem.

24. Show that a function continuous on $[a, b]$ is R-integrable on $[a, b]$.

25. Show that a function which is continuous on a closed bounded interval is also uniformly continuous on that interval.

26. Show that continuity is a necessary condition for the existence of a finite derivative but not a sufficient condition.

27. Show that the set of rational numbers Q does not satisfy the Completeness property.

28. Show that the sequence (s_n) defined by :

$$s_n = \left\{\sqrt{n+1} - \sqrt{n}\right\}, \forall\, n \in \mathrm{N} \text{ is convergent.}$$

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*

Each question carries 10 marks.

29. Show that if $f \in \mathrm{R}[a, b]$ then $|f| \in \mathrm{R}[a, b]$. and $\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$.

30. A sequence (s_n) converges iff it is a Cauchy's sequence.

31. (i) If a function f is continuous on a closed and bounded interval $[a, b]$ then it attains its bound on $[a, b]$.

(ii) Test the continuity of the function $f(x)$ at $x = 0$ where

$$f(x) = x \sin\left(\frac{1}{x}\right), \quad x \neq 0;$$
$$= 0 \qquad \text{at } x = 0.$$

32. State and Prove second fundamental theorem on integral calculus.

(2 × 10 = 20 marks)

S70360

V SEMESTER B.Sc. DEGREE EXAMINATION, Nov 2017

(CUCBCSS-UG)

Statistics

STS 5B 05 –MATHEMATICAL METHODS IN STATISTICS

Time: 3 Hours

Maximum: 80 Marks

Section A - Answer all questions in one word. Each question carries 1 mark.

1. 2/3
2. $(1+x)^n \geq 1+nx$
3. 1
4. Sup(A,B)
5. .it is bounded.
6. l.
7. F(c)(b-a)
8. True
9. False
10. true
11. proof of triangular inequality
12. (S_{2n}) and (S_{2n-1}) are convergent hence(S_n)convergent
13. Proof 2 marks
14. Proof 2 marks
15. Proof 2 marks
16. Proof 2marks
17. Proof 2marks
18. Proof 4 marks
19. Proof 4 marks
20. Proof 4 marks
21. $C=2\pm 2/\sqrt{3}$
22. Proof 4 marks
23. Cauchys mean value theorem statement 2 marks proof 4 marks
24. Proof 6 marks
25. proof6 marks
26. Proof 6 marks
27. Proof 6 marks
28. $Sn=1/2\sqrt{n}$ $|Sn-0|<e$ if all n>m converges to 0
29. Proof 10 marks
30. Proof 10 marks Cauchy sequence converges.
31. (a) proof (b) F(0)=0,f(0+)= 0, f(0-)=0 continuous.
32. Statement 2 marks proof 8 marks.